

# ABANDONING MONOMORPHISMS: PARTIAL MAPS, FRACTIONS AND FACTORIZATIONS

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ABSTRACT. For a composition-closed and pullback-stable class  $\mathcal{S}$  of morphisms in a category  $\mathcal{C}$  containing all isomorphisms, we form the category of  $\mathcal{S}$ -spans  $(s, f)$  in  $\mathcal{C}$  with first “leg”  $s$  lying in  $\mathcal{S}$  and, as its quotient category, give an alternative construction of the category  $\mathcal{C}[\mathcal{S}^{-1}]$  of  $\mathcal{S}$ -fractions, the intermediate steps of which are of independent interest. Instead of trying to turn  $\mathcal{S}$ -morphisms “directly” into isomorphisms, we turn them separately into retractions and into sections, in a universal manner. The second of these two quotient processes leads to a legitimate candidate for playing the role of the  $\mathcal{S}$ -partial map category when  $\mathcal{S}$  is not constrained to contain only monomorphisms of  $\mathcal{C}$ . Under mild additional hypotheses on  $\mathcal{S}$ , but still without the mono constraint, this  $\mathcal{S}$ -partial map category has a localization, which is a split restriction category (in the sense of Cockett and Lack), or even a split range category (in the sense of Cockett, Guo and Hofstra), and which is still large enough to have  $\mathcal{C}[\mathcal{S}^{-1}]$  as its quotient. The construction of the range category is part of a global adjunction between relatively stable factorization systems and split range categories.

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